**Simulation of Brownian Motion – Laboratory nr 1**

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Date: 11.03.2024

1. **Introduction**

Brownian motion is the random motion of the particles suspended in the fluid, which was first observed by the scottish botanist Robert Brown in 1827 and first explained by Albert Einstein in 1905. This explanation of Brownian motion served as convincing evidence that atoms and molecules exist, and was further verified experimentally by Jean Perrin in 1908. Perrin was awarded the Nobel Prize in Physics in 1926 "for his work on the discontinuous structure of matter". The direction of the force of atomic bombardment is constantly changing, and at different times the particle is hit more on one side than another, leading to the seemingly random nature of the motion.

1. **Execution of the exercise**

**2.1 Creating simulation**

At first we need to create a program to simulate brownian motion. Programming language used in the simulation is Python 3.10. Class „Particle” was created to simulate single particle movement:

import numpy as np

class Particle:

    def \_\_init\_\_(self, x=0, y=0):

        self.x = x

        self.y = y

        self.lista\_x = [x]

        self.lista\_y = [y]

    def make\_step(self):

        self.x += np.random.normal(0, 1, 1)[0]

        self.y += np.random.normal(0, 1, 1)[0]

        self.lista\_x.append(self.x)

        self.lista\_y.append(self.y)

**2.2 Executing simulation**

Next, the movement of single, ten, hundred and thousand particles was presented. Each of these plots presents particles after 100 iterations.

Obraz zawierający tekst, diagram, mapa

Opis wygenerowany automatycznie

Obraz zawierający tekst, diagram, mapa

Opis wygenerowany automatycznie

Obraz zawierający tekst, zrzut ekranu, diagram, mapa

Opis wygenerowany automatycznie

Obraz zawierający tekst, zrzut ekranu, mapa, Wielobarwność

Opis wygenerowany automatycznie

**2.3 Mean square displacement**

Next, on the basis of trajectory of thousand particles, mean square displacement was calculated for each iteration. Results are presented below:

Obraz zawierający tekst, linia, Wykres, diagram

Opis wygenerowany automatycznie

Obraz zawierający tekst, zrzut ekranu, linia, Wykres

Opis wygenerowany automatycznie

Coefficient of fitted line was calculated as of: 2.070

**2.4 Autocorellation**

Next thing that was done, was calculating autocorellation for the single trajectory of a particle. Randomly generated data don’t posses this trait, but brownian particle trajectory is autocorellated.

Obraz zawierający tekst, diagram, zrzut ekranu, Wykres

Opis wygenerowany automatycznie

Obraz zawierający tekst, diagram, linia, Czcionka

Opis wygenerowany automatycznie

Obraz zawierający tekst, linia, zrzut ekranu, Wykres

Opis wygenerowany automatycznie

We see the expected rusults on the above plots.

**2.5 Density plot**

Lastly, the heatmap of density points was created and evaluated over time.

Obraz zawierający diagram, zrzut ekranu, piksel

Opis wygenerowany automatycznieObraz zawierający diagram, zrzut ekranu, piksel

Opis wygenerowany automatycznie

Obraz zawierający diagram, zrzut ekranu, piksel

Opis wygenerowany automatycznie

On the histograms we see that X&Y coordiantes and density of the particles is being shifted towards the rim, but still the most dense point on the Surface is starting point (0, 0).

1. **Conclusion**

Brownian motion is not solely based on random distribution (as implemented in the model), but motion on the basis of bell curve distribution is excellent for the sake of simulation. With ‘random walk’ we can describe and predict diffrent subject in the fields of physics, chemistry, economics, psychology and other related sciences.

From the conducted laboratory we can conclude few things: ‘random walk’ is an acceptable aproximation of brownian motion, brownian motion is random on the scale of one particle, but on the larger scale submits to the rules (such as mean square distance dependant on time, iterations), brownian motion is in fact autocorellated in contrast to random generated data.